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Quantum Measurements, Phase Transitions, and Spontaneous Symmetry Breaking

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It is shown that if quantum measurement results in a phase transition in a detector medium, then due to orthogonality of Fock spaces of different vacuums the final state cannot be distinguished from the mixture. Examples for transitions in ferromagnetic and vapor–liquid systems are considered. A new formalism is proposed in which EPR-Bohm nonlocal correlations are described as spontaneous symmetry breaking of the pair system state.

The analogy between quantum measurement and the spontaneous symmetry breaking (SSB) mechanism of nonperturbative quantum field theory (NQFT) was pointed out first by Neeman (1986). Besides gauge field theories, the phenomenon of SSB is well known for macroscopic phase transitions. Some of these are applied for the detection of elementary particles, for example, liquid–vapor transitions in Wilson and bubble chambers. In standard quantum mechanics (QM) particle interaction with a detector medium during the measurement of an observable l should result in the entanglement of the quantum states of particle g and detector S

$$|g_0\rangle|S_0\rangle \to \sum C_i|l_i\rangle|S_i\rangle \tag{1}$$

So in principle interference between different states $|l_i\rangle$, $|s_j\rangle$ can be observed, which is impossible for mixed states as predicted by the QM reduction postulate (RP).

First we regard qualitatively the measurement of the coordinates of a relativistic particle g in a Wilson chamber filled with saturated vapor. We take $|g\rangle$ as a superposition of two flight paths $|x_1\rangle$ inside and $|x_2\rangle$ outside the chamber. If g passes through the vapor, it can transfer high momentum

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to one of the atomic electrons, which will leave the chamber practically without secondary interactions. At the interaction point a positive ion will be left at rest which will attract electrostatically surrounding atoms due to their multipole moments. If the vapor pressure is higher than the saturation pressure, this will result in the formation of a liquid drop which can grow in size unrestrictedly, corresponding to a phase transition of the first kind (Landau and Lifshitz, 1958). It is impossible now to solve the evolution equations for such a state. Only final system parameters can be obtained from the study of the minimum of the thermodynamic potential for a two-phase system. The formation of the new phase is checked by the approximation of the drop surface free energy to its normal macroscopic value. It occurs when the drop radius is $R >> 10^{-6}$ cm, which corresponds to 10^{4-6} atoms in it. Interference effects are expected to be proportional to nondiagonal terms of the operator $\hat{M} = \hat{M}_A \cdot \hat{M}_S$ where $\hat{M}_A$ and $\hat{M}_S$ are particle and detector observable measurements, respectively. So we must consider terms of $\overline{M} = \mathrm{Tr}(\rho_f \hat{M})$ of the type $\langle x_1 | \hat{M}_A | x_2 \rangle \cdot \langle L_l | \hat{M}_S | V_J \rangle$. But due to the orthogonality of Fock spaces of different phases, M_S matrix elements should be zeros. This occurs for any linear operator M_S which could be written as a sum of finite products of annihilation–creation operators of one-phase excitations a_i, a_i^+ (quasiparticles) and so cannot connect states from different phases. For the vapor the Fock ground state is the normal vacuum and for the liquid the new vacuum is its ground state and quasiparticles are phonons. So if all practically measurable operators are of M_S type, this interference is unobservable.

Now we regard phase transitions of the second kind, where some approximate calculations are possible for ferromagnetic media. We consider the measurement of the charged particles coherent state $|A\rangle = \alpha_1 |x\rangle + \alpha_2 |x_2\rangle$ of two parallel paths $|x_{1,2}\rangle$ passing along opposite sides of a small ferromagnetic sample FS whose initial magnetization is negligible. This can be approximately fulfilled: (a) if the sample is in the paramagnetic state at $T > T_c(P)$, and instantaneously by the change of pressure it transfers into the ferromagnetic phase, whose magnetization is defined by the external field; (b) for soft magnetic materials for $T << T_c$, when many domain fields equalize each other. The Hamiltonian of FS in the external (particle's) field $\mathbf{H}$ is

$$\hat{H}_F = -\sum b_{ij} \mathbf{S}_i \mathbf{S}_j - M \sum \mathbf{H} \mathbf{S}_j \tag{2}$$

where b_{ij} is the atomic exchange integral. The change of spin systems S^2 and S_z is allowed due to interaction with the crystal lattice orbital momentum, which is omitted in (2). We take the particle's weak magnetic field to be nearly constant in the FS volume and directed along the z axis. For the path superposition the field is the operator

$$\hat{H} = \mathbf{H}_+ + \mathbf{H}_- = \sum_{i=1,2} \frac{q[\hat{\mathbf{p}} \cdot \mathbf{R}_i]}{r_i^3} \tag{3}$$

where q is the charge, $\mathbf{p}$ is the momentum operator of the particle, and $\mathbf{r}_{1,2}$ are the radius vectors from the two particle paths to the FS center. Also it is supposed that the time of flight when $\mathbf{H}$ acts on FS is $\tau_F >> \tau_T$, the phase transition time, which for the ferromagnetic phase is of the order 10^{-3} sec. Under these conditions it can be shown that for both examples (a) and (b) the minimum of the Hamiltonian H_F density achieved in the entangled state is

$$|\Psi_f\rangle = \frac{1}{\sqrt{2}} (\alpha_1 |x_1\rangle |U_F\rangle + \alpha_2 |x_2\rangle |D_F\rangle) \tag{4}$$

which survives even after $\mathbf{H}$ is turned off. $|U_F\rangle = |u_1\rangle \cdot |u_2\rangle \ldots$ is the FS spin state polarized along the z axis and $|D_F\rangle$ is that in the opposite direction. $|U_F\rangle$ and $|D_F\rangle$ are two different vacuum states of the ferromagnetic whose excitations (magnons) are also orthogonal, which we denote as $|k_u\rangle$, $|k_d\rangle$. In the NQFT framework, as no selection rules prohibit it, with FS initially in an unspecified pure or mixed state, the system asymptotically will evolve to the single pure state (4) in which the Hamiltonian (2) reaches its minimum (Itzykson and Zuber, 1979). As in the previous example, any nondiagonal terms are infinitely small for the polynomial of a_k, a_k^+, and $S + A$ interference is unobservable in this case. For example, this is true for the magnetization $\mathbf{I}$ which defines the FS induction, because $|\mathbf{I}| = N - \Sigma\, a_k a_k^+$. This result is expected to be true approximately in the temperature range $T < 0.3T_c$ where the relative weight of thermal magnon excitations in the density matrix is of the order 10^{-2} relative to the vacuum state weight (Landau and Lifshitz, 1958). Thermal excitations must gain decoherence additionally, which is not accounted for here.

Preliminary NQFT path integral calculations of Ward–Takahasi identities for spin field Ψ with spin operators S^i and the $SU(2)$-invariant Lagrangian $L(\Psi) = L(e^{i\alpha_i \sigma_i}\Psi)$ for $i = 1, 2, 3$ support those results. A small noninvariant perturbation $S^3\hat{H}$ from the external fields of (3) is to be added to $L(\Psi)$. The resulting generating functional $W[J]$, where $J(x)$ is an arbitrary function, is invariant under $SU(2)$ rotations $\partial W/\partial\alpha_i = 0$ (Itzykson and Zuber, 1979). The subsequent functional derivatives $\delta W/\delta J$ result in the equation $\langle S^3(x)\rangle = \int d^4y\, \langle \hat{H} S^i(y) S^i(x)\rangle$ for $i = 1, 2$, where the expectation is taken on both spin vacuum and $|A\rangle$ variables, and whose solution coincides with (4). It was shown that the same breakdown can be induced also for restricted field volume or finite N, changing only the magnon effective mass and minimal field limit required for the SSB initiation (Celeghini *et al.*, 1990).

The final state (4) is the same as for the Koleman–Hepp (KH) model for an ensemble E_S of $N < \infty$ polarized free spins (Hepp, 1972). It was shown that in this case for the observable $\hat{Q} = \Pi^N (\sigma_{xi})$ the nondiagonal matrix element A_Q is large (Bell, 1975),

$$A_Q = \langle U_F|\hat{Q}|D_F\rangle \sim |\langle u_i|\sigma_{zi}|u_i\rangle|^N$$

To measure Q, each particle of E_S should be sent through a Stern–Gerlach magnet where each S_X is measured separately. For FS states no measurement on the FS as a whole body can help to define the Q value, because all FS observables are polynomials of a_k, a_k^+ of rate n less than 4 (Ziman, 1964). Note that $\hat{H}_F$ is only the electron exchange part of the full lattice Hamiltonian $\hat{H}_{em}$. To measure $\hat{Q}$ by use of a Stern–Gerlach magnet we must first to break the FS lattice into atoms, not disturbing the spin collective state, which is obviously impossible. Even if a single atom is removed from the lattice by projectile scattering or some external field, this will result in magnon excitations of the lattice and distort the Fock space basis. Even if this can be performed coherently when only $N < 10^{5-6}$ atoms are left in the lattice, it becomes paramagnetic due to additional electron momentum acquired in a small volume due to the uncertainty relation (Strikman and Treves, 1963). So further spin measurement loses any sense.

This measurability restriction for individual particles seems quite natural for the secondarily quantized states originating from the collective interactions. It seems that all the observables which can be really measured for FS, for example, the magnetization $\mathbf{I} = \Sigma^N \boldsymbol{\sigma}_i$, behave as classic stochastic values. This means that our set of FS observables is described by a C^*-algebra, which introduces restrictions of measurability analogous to classic meter-pointer observables (Hepp, 1972). This change in comparison with the KH model in the N-finite case is explained by the presence of the electromagnetic field with infinite number of degrees of freedom. Its infrared divergencies supposedly define the SSB mechanism in ferromagnetics (Celeghini *et al.*, 1990).

Now we study possible connections of state vector symmetry breaking with space-time symmetries as in the gauge field theories. Above, the value of the SSB parameter—for example, the direction of magnetization—was purely stochastic. In the quantum mechanical formalism if two particles have interacted previously, then some of their measured observables are correlated. These nonlocal correlations are defined by the QM reduction postulate (RP).

For the EPR-Bohm singlet two-fermion system the final state after S_Z measurements on both particles is

$$\Psi_f = \frac{1}{\sqrt{2}}(|S_u^1\rangle|u\rangle_1|d\rangle_2|S_d^2\rangle - |S_d^1\rangle|d\rangle_1|u\rangle^2|S_4^2\rangle) \tag{5}$$

where $S_u^1, \ldots$ are the states of detectors D_1, D_2 for u, d states of particles.

For continuous observables, arbitrary hydrogenlike potential systems $M = A_1 + A_2$ which can be free or bound to some point are studied. Their wave function can be written in general

$$|M\rangle = \psi(r_1, r_2) = \chi(R)\phi(r_{12}) \tag{6}$$

where R_1, r_2 are coordinate vectors of particles A_1, A_2; $r_{12} = r_1 - r_2$; and $R = (m_1r_1 + m_2r_2)/(m_1 + m_2)$ is the system center-of-mass coordinate. For a free M system, $\chi(R) = c\ \exp(ipR)$, where p is the total momentum of M, and M is uniformly distributed inside the large volume H^3 with dimensions $\sim H$. We also assume M has a finite size, characterized by $R_M = 0.5|r_1 - r_2|$ and $\varphi(r) \Rightarrow 0$ at $r >> R_M$.

R and r_{12} are M generalized coordinates (GC) and in terms of them $|M\rangle$ is factorized into the wave function χ of the system M as a whole and ϕ describing the relation between its parts. If idealized detectors, without disturbing another particle, measure instantaneously at r_1 and r_2, their correlations must be measured. In principle such r_1, r_2 correlations can be detected if M is unstable and decays in flight at the moment t and A_1, A_2 have very low velocities. At $t_1 = t + dt$ we insert in H^3 detectors D_1, D_2 and measure r_1, r_2. The wave packet smearing is small for small dt, so we find $|r_1 - r_2| <<$ H and $R = 0.5(r_1 - r_2)$. This correlation can be regarded as induced by R reduction to the same value in both r_1 and r_2 measurements, when $\phi(r_{12}) = \delta(r_{12})$ relative to the H scale. It is supposed that nonlocal correlations can be connected with the fictitious state vector dependence on one GC which is reduced to the same unique value in both measurements.

For the EPR-Bohm singlet pair its spin state vector can be rewritten through the GC $T_z = S_{z_1} - S_{z_2}$, $J_z = S_{z_1} + S_{z_2}$; we have

$$|V_{12}\rangle = 1/\sqrt{2}(|T_z = 1\rangle - |T_z = -1\rangle)\delta(J_Z) \tag{7}$$

In fact this is a function of one variable describing system V_{12} electron spin orientation in space for $J_z = 0$, $J^2 = 0$, which are also GC. Obviously this approach agrees with the Bell theorem, because it conserves amplitudes structure, despite the transformation of observables. For example, for a spin 1/2 pair, if the angle between z_1 and z_2 is α, calculation for the amplitudes of T_z, J_z gives the expectation value for projection on z_1, z_2 to be $E(z_1, z_2) = -\cos\alpha$ (Bell, 1975).

This formalism leads to a new GC interpretation of quantum nonlocality derived from the supposed hierarchy of system observables. We regard on an equal basis the macroscopic world W and any free quantum systems M_l constituting the set of independent quantum systems S_q. We suppose that in nature there are really fundamental quantum observables which describe relations between such S_q or S_q internal coordinates. Such observables are to be unambiguous despite their initial uncertainty and consequently are reduced to the same value being measured at separate space points. Individual particle observables are derivatives of such primary system GC which result in their distant correlations. For system M if r_1 or r_2 is measured, the reduction of

R occurs and due to it r_1, r_2 correlations are observed. The same approach is applicable for spin space, where the EPR-Bohm singlet state can be regarded as independent of the external world W and T_z describes their relative orientation. In classical physics any choice of system GC is equivalent, but in quantum mechanics the preferable GC is revealed by the reduction of the correlated states.

It is well known that the standard RP in which state vector reduction is to occur instantaneously in all space formally is not covariant. The covariant RP can be formulated only for observer-dependent (OD) wave functions $\Psi_O(t, x_O, x_1, d_1, x_2, d_2, \ldots)$, where x_O and t are observer position and time axis, and x_I, d_I are particle coordinates and other observables (Finkelstain, 1992). After the measurement at x_1, t the reduced wave function Ψ'_O will exist for x_O inside the x_1, t_1 future light cone and initial Ψ_O will exist outside of it. The proposed ODRP is covariant, as the light cone transforms into itself and satisfies quantum field causality.

Now we consider RP for a correlated state which can be described by the GC set D_I. For this purpose we take the former OD function for which $d_J = F(D_I)$ and transform it into OD $\Psi_O(t, x_O, D_I)$. In case of correlated measurements at x_1, x_2 the ODRP yields that the reduced function $\Psi'_O(D_I)$ will exist inside two light cones starting at x_1, t and x_2, t and $\Psi_O(D_I)$ stays outside of it. It is supposed that spacelike-separated measurements at x_1 and x_2 can be regarded as independent of each other in the sense that the measurement at x_1 does not influence the result of the measurement at x_2 and vice versa. This independence supposedly is a consequence of the symmetry of the measurements at x_1 and x_2 under Lorentz transformations, in which one event can become the past or the future relative to another. It fits into the GC nonlocality interpretation, which derives correlations from the reduction of fundamental observables describing relations of independent quantum systems to a unique value at the different space points.

These results suppose that the correlation between the two measuring devices which measure parameters of the EPR pair can be interpreted also as the global (nonlocal) SSB of the system state which is broken by these measurements. This reflects the fact that the same observable $T_z = S_{z_1} - S_{z_2}$ can be taken as the SSB parameter for the two devices in formula (5). So in this case two space-separated phase transitions in these devices will evolve in correlation defined by this parameter.

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